



MATHEMATICS

MAXIMUM MARKS: 80

TIME ALLOWED: THREE HOURS

(CANDIDATES ARE ALLOWED ADDITIONAL 15 MINUTES FOR ONLY READING THE PAPER. THEY MUST NOT START WRITING DURING THIS TIME.)

- *This Question Paper consists of three sections A, B and C.*
- *Candidates are required to attempt all questions from Section A and all questions EITHER from Section B OR Section C.*
- *Section A: Internal choice has been provided in two questions of two marks each, two questions of four marks each and two questions of six marks each.*
- *Section B: Internal choice has been provided in one question of two marks and one question of four marks.*
- *Section C: Internal choice has been provided in one question of two marks and one question of four marks.*
- *All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.*
- *The intended marks for questions or parts of questions are given in brackets [].*
- *Mathematical tables and graph papers are provided.*

SECTION A - 65 MARKS

Question -1

(i) If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = (1+x)(1-x)$, then $f(A)$ is

(a) $-4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(b) $-8 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $4 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(d) None of these

(ii) If $\int \frac{5 \tan x}{\tan x - 2} dx = ax + b \log |\sin x - 2 \cos x| + C$, find the value of a and b.

(a) $a=1, b=2$

(b) $a=2, b=3$

(c) $a=0, b=5$

(d) $a=9, b=4$

(iii) The value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{-\pi}{3}$

(d) $\frac{\pi}{4}$

(iv) $\frac{dy}{dx} = \frac{x^2y + y^2x}{x^n + y^n}$ is

(a) $n=3$

(b) $n=5$

(c) $n=-2$

(d) $n=4$

(v) Let A and B 2 students such that $P(A) = \frac{1}{2}, P(B) = P \& P(A \cup B) = \frac{3}{5}$, find 'P' if A and B are independent events.

(a) $P = \frac{1}{5}$

(b) $P = \frac{1}{3}$

(c) $P = \frac{2}{5}$

(d) None of these

(vi) If $A = \begin{bmatrix} x+4 & 2 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} x & 2x-1 \\ 2 & 5 \end{bmatrix}$ and $|AB| = 12$, find the value(s) of x.

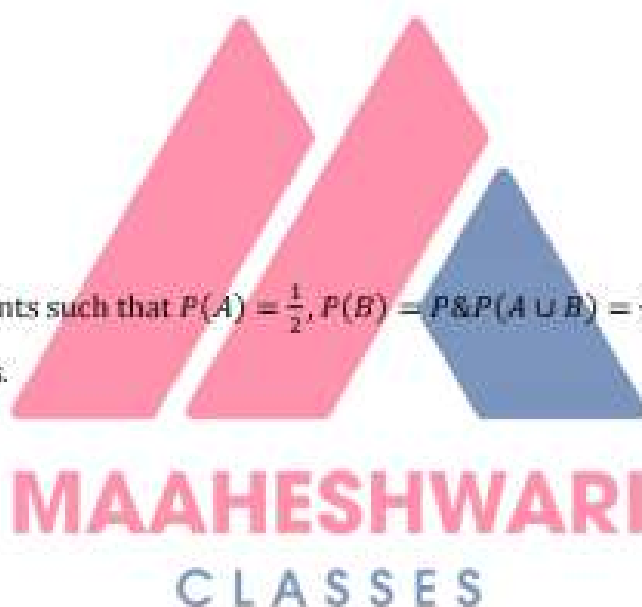
(a) $x=4, -4$

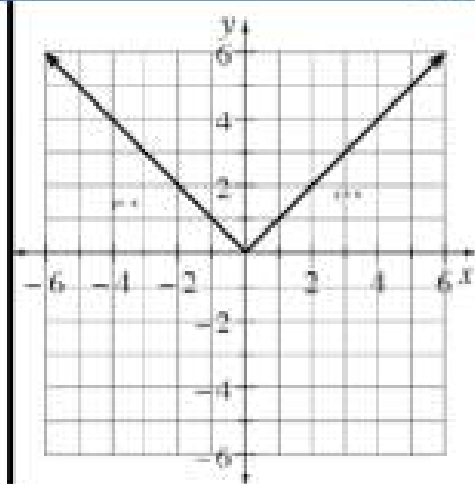
(b) $x=2, 4$

(c) $x=3, 6$

(d) $x=2, 2$

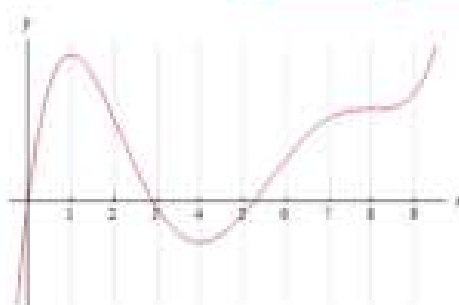
(vii) Identify the function that the following graph represents.





- (a) Identify function
- (b) Polynomial function
- (c) Rational function
- (d) Modulus function

(viii) Determine the intervals on which the following function increases.



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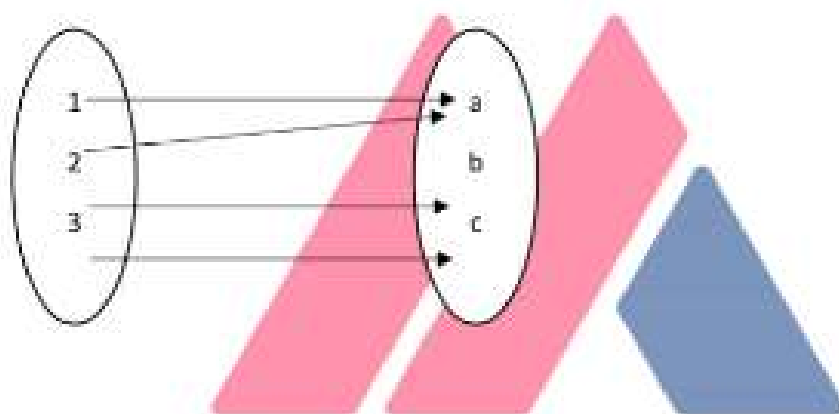
- (a) $(-\infty, 1), (4, 8) \& (8, \infty)$
 - (b) $(-\infty, 1), (4, 8) \& (-8, -\infty)$
 - (c) $(-\infty, -1), (4, 8) \& (8, \infty)$
 - (d) $(-\infty, 1), (4, 8) \& (8, \infty)$
- (ix) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = x^2$. Then, f is
- (a) Both one-one and onto
 - (b) One-one but not onto
 - (c) Onto but not one-one
 - (d) Neither one-one nor onto

(x) Assertion (A) : $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a diagonal matrix.

Reason (R) : A square matrix is called a diagonal matrix if all the non-diagonal elements are zero and all its diagonal elements are non-zero.

- (a) Both A and R are correct and R is the correct explanation of A.
- (b) Both A and R are correct but R is not the correct explanation of A.
- (c) A is correct but R is not correct.
- (d) A is not correct but R is correct.

(xi) Check whether the below represented figure shows an onto function or not. Give reason.



(xii) If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$, find x and y so that $A^2 + xI_2 = yA$.

(xiii) Consider the non-empty set consisting of exactly two boys in a family and a relation R defined as $a R b$ if a is brother of b . Then R is

- (a) Reflexive
- (b) Symmetric
- (c) Transitive
- (d) None of these

(xiv) Three cards are drawn at random from a pack of 52 cards. What is the probability that the drawn cards are all kings?

(xv) An experiment yields 3 mutually exclusive and exhaustive events A , B and C . If $P(A)=2$, $P(B)=3P(C)$, then $P(A)$ is equal to

- (a) $\frac{1}{11}$
- (b) $\frac{2}{11}$
- (c) $\frac{3}{11}$
- (d) $\frac{6}{11}$

Question -2

- (i) The table below shows some of the values of differentiable functions u and v and their derivatives.

x	$u(x)$	$u'(x)$	$v(x)$	$v'(x)$
1	0	2	-1	5
2	4	5	3	2
3	2	-1	3	0

If $p(x) = u(v(x))$ find $p'(3)$.

OR

- (ii) The volume of a cube is increasing at the rate of $9 \text{ cm}^3/\text{s}$. How fast is the surface area increasing when the length of an edge is 10 cm ?

Question -3

Write the value of $\int_0^1 \frac{e^x}{1+e^{2x}} dx$.

Question -4

Find a point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining the points $(2, 0)$ and $(4, 4)$.

Question -5

- (i) Evaluate the integral: $\int \frac{x+2}{\sqrt{3x+1}} dx$.

- (ii) Evaluate the integral: $\int \log|x| dx$.

Question -6

Find the domain of the following function: $f(x) = \frac{1}{\sqrt{x+|x|}}$

Question -7

Prove that $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left(\frac{1-x}{1+x} \right)$.

Question -8

Evaluate: $\int \frac{5x-2}{1+2x+3x^2} dx$.

Question -9

- (i) If the function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ is derivable, then find the values of a and b .

(ii) If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$

Question -10

Three friends roll a fair six-sided die each. If one person's number is different from the other two (i.e., two get same number, third different), that person pays. Otherwise, they roll again.

- (a) Probability in one round that two get the same number and the third different?
(b) Probability they need exactly two rounds?

OR

A company has three factories:

- Factory X makes 50% of items, defect rate 1%
- Factory Y makes 30% of items, defect rate 2%
- Factory Z makes 20% of items, defect rate 5%

An item is found defective. (a) Define events.

(b) Find probability it came from Factory Z.

(c) Which factory is most likely the source of a defective item?

Question -11

The sum of three numbers is 6. If we multiply third number by 3 and add second to it we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Question -12

(i) Find the particular solution of the differential equation: $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$ given that $y = 1$ where $x = 0$

(ii) Evaluate $\int \sqrt{\tan x} \, dx$.

Question -13

- (i) A closed rectangular box is made of aluminium sheet of negligible thickness and the length of the box is twice its breadth. If the capacity of the box is 243cm^3 , compute the dimensions of the box of least surface area.
- (ii) Find the dimensions of the rectangle of perimeter 36cm which will sweep out a volume as large as possible, when revolved about one of its sides. Also find the maximum volume.

Question -14



There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean of X .

Section B 15 MARKS

Question -15

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) If the vectors $a\hat{i} + 3\hat{j} - 2\hat{k}$ and $3\hat{i} - 4\hat{j} + b\hat{k}$ are collinear, then $(a, b) =$
(a) $(\frac{9}{4}, \frac{8}{3})$ (b) $(-\frac{9}{4}, \frac{8}{3})$ (c) $(\frac{9}{4}, -\frac{8}{3})$ (d) $(-\frac{9}{4}, -\frac{8}{3})$
- (ii) Find the equation of the plane passing through $(-2, 1, 3)$ and perpendicular to the line having direction ratios $\langle 3, 1, 5 \rangle$.
- (iii) Find the angle between the vectors $\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 9\hat{j} + 6\hat{k}$.
- (iv) The intercepts made by the plane $3x - 2y + 4z = 12$ on the coordinates axes are
(a) 6, -4, 3 (b) 4, -6, 3 (c) 2, -3, 4 (d) $\frac{1}{4}, -\frac{1}{6}, \frac{1}{3}$
- (v) Find the volume of the parallelepiped with co-terminous edges $2\hat{i} + \hat{j} - \hat{k}$, $\hat{i} + 2\hat{j} + 3\hat{k}$ and $-3\hat{i} - \hat{j} + \hat{k}$.

Question -16

- (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, find $(\vec{r} \times \hat{i}) \cdot (\vec{r} \times \hat{j}) + xy$.

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- (b) Using vectors, find the value of k such that $(k, -10, 3)$, $(1, -1, 3)$ and $(3, 5, 3)$ are collinear.

Question -17

- (a) Find the shortest distance between the lines $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = 2\hat{i} + 4\hat{j} + 5\hat{k} + \mu(4\hat{i} + 6\hat{j} + 8\hat{k})$.

OR

- (b) Find the angle between the line $\hat{i} + 4\hat{j} + 5\hat{k}$

Question -18

Find the area of region bounded by the curve $y = 6x - x^2$ and $y = x^2 - 2x$.

SECTION C 15 MARKS

Question -19

In- sub-parts(i) and (ii) choose the correct options and in sub-parts (iii) to (v) , answer the questions as instructed.

- (i) Which relation is correct for breakeven point where $R(X)$ is revenue function and $C(x)$ is cost fnction?

(a) $R(X) > C(X)$ (b) $R(X) < C(X)$ (c) $R(X)=C(X)$ (d) $R(X) = 2C(X)$
- (ii) If the lines of regression are parallel to coordinate axes, then the coefficient of correlation is :

(a) 1 (b) 0 (c) ∞ (d) $\frac{1}{2}$
- (iii) For a given bi-variate distribution, the mean of variable $x = 4$ and the mean of variable $y = 6$. Find the point of intersection of two regression lines.
- (iv) Average revenue of a commodity is given by $AR(x) = a + \frac{b}{x}$. Find the demand function when marginal revenue is zero.
- (v) If $R(X) = 36x + 3x^2 + 5$ then find actual revenue from selling 50th item.

Question -20

- (i) The total revenue function $R(x) = a + 2x^3 - 3.5x^2$. find the point where marginal revenue curve cuts the co-ordinate axis.
- (ii) For the revenue function $R(x) = \frac{b}{a+x}$ show that the marginal revenue function is increasing for all $b < 0$ and $a > 0$.

Question -21

The line of regression of marks in Math (X) and marks in English (Y) for a class of 50 students is $3Y - 5X + 180 = 0$. The average score in English is 44 and variance of marks in Maths is $\frac{9}{16}$ th of the variance of marks in English. find the average score in Maths. Also, find out the coefficient of correlation between marks in Maths and English.

Question -22

- (i) Solve the following linear programming problem graphically and interpret your result of $Z = 2x - 5y$ subjects the constraints $x + y \geq 2$, $x - y \geq 0$, $x \leq 2$, $x \geq 0$, $y \geq 0$.

OR

- (ii) The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients B_1 and B_2 costs Rs.5 per kg and B_2 costs Rs.8 per kg. Strength considerations dictate that the brick should not contain more than 4 of B_1 and minimum 2 kg of B_2 . Since the demand for the product is likely to be related to the price of the bricks find the minimum cost of the brick satisfying the above conditions. Formulate this situation as an L.P.P. and solve it graphically.

